MODEL: *D*

KING ABDULAZIZ UNIVERSITY

DEPARTMENT OF MATHEMATICS

Exam/Course: Final Exam - Math-204

Student Name: **Student University Number:**

Instructor Name: **Section:**

Time Allowed: 120 Minutes **Jan. 20, 2011**

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($Q1$) **Select the correct response:**

(i) The D.E. $x\frac{dy}{dx} = y\,e^{\frac{x}{y}} - x$ is

☐ exact ☐ Ricatti ☐ homogeneous ($2Pt.$)

(ii) The D.E. $(x+1)\frac{dy}{dx} = y - 1 + x^2$ is

☐ linear ☐ Bernoulli ☐ separable ($2Pt.$)

(iii) $y = \frac{\pi}{2}$ is the unique solution of (IVP): $y' + 2x\cos y = 0;\ y(1) = \frac{\pi}{2}$

☐ true ☐ false ($2Pt.$)

(iv) There is a particular solution of $y' + P(x)y = Q(x)$ in the form $\int Q(t)e^{\int P(t)dt}dt$

☐ true ☐ false ($2Pt.$)

(v) The function $f(t) = t^{-2}$ **is not** piecewise continuous

☐ true ☐ false ($2Pt.$)

(vi) The function $f(t) = \sin t$ **is not** of exponential order

☐ true ☐ false ($2Pt.$)

(vii) The function $F(s) = \ln(\frac{s}{s+4})$ **is** the Laplace transform of a function that is piecewise continuous and of exponential order

☐ true ☐ false ($2Pt.$)

($viii$) $\ell^{-1}\{F(s)G(s)\} = f(t)g(t)$

☐ true ☐ false ($2Pt.$)

(Q_2) A mass weighing 8 pounds is attached to a 7-feet-long spring. At equilibrium the spring measures 3 feet. If the mass is initially released from the rest at a point 2 feet below the equilibrium position. Find the displacements $x(t)$ if it is further known that the surrounding medium offers a resistance numerically equal to $\frac{3}{2}$ times the instantaneous velocity. ($10Pt.$)

(Q_3) **Find the general solution of:** $y''-2y'+y = e^t sin^{-1}t$ ($10Pt.$)

(Q_4) **Find the general solution of:** $y(x-y+1)dx+(x-2y)dy=0$ ($8Pt.$)

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(Q_5) **Find the Laplace transform**: (i) $\ell\{te^{-2t}\sin 4t\}$, (ii) $\ell\{sint\,u(t-\frac{\pi}{2})\}$ ($8Pt.$)

(Q_6) **Find the inverse Laplace transform**: (i) $\ell^{-1}\{\frac{1}{(s^2+1)^2}\}$, (ii) $\ell^{-1}\{\frac{e^{-\pi s}}{s^2+2s+2}\}$ $(10Pt.)$

(Q_7) **Use Laplace transform to solve:** $y''+y = g(t);\ y(0) = 1,\ y'(0) = 2,$ $(10Pt.)$

$$g(t) = \begin{cases} 0 & \text{if } 0 \leq t \leq \frac{\pi}{2}, \\ \sin t & \text{if } t > \frac{\pi}{2}. \end{cases}$$

Answer only one of the following two questions:

(Q_8) **Solve:** $\frac{dy}{dx} = \frac{1-y^2}{1-x^2}$; $y(0) = 0$ ($8Pt.$)

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(Q_9) **Solve:** $\frac{dy}{dx} = xy^{-\frac{1}{2}} - y$; $y(0) = 9$ ($8Pt.$)

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Sum	Bal.