MODEL: C

KING ABDULAZIZ UNIVERSITY

DEPARTMENT OF MATHEMATICS

Exam/Course: Final Exam - Math-204

Student Name: **Student University Number:**

Instructor Name: **Section:**

Time Allowed: 120 Minutes **Jan. 20, 2011**

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$(Q1)$ **Select the correct response:**

(i) The D.E. $\frac{dy}{dx} = \frac{x-y}{x^2}$ is

☐ linear ☐ Bernoulli ☐ separable $(2Pt.)$

(ii) The D.E. $\frac{dy}{dx} = \frac{x}{y} + \frac{y}{x} + 1$ is

☐ exact ☐ Ricatti ☐ homogeneous $(2Pt.)$

(iii) $y = \frac{1}{x^2}$ is the unique solution of (IVP): $y' + 2xy^2 = 0;\ y(-1) = 1$

☐ true ☐ false $(2Pt.)$

(iv) There is a particular solution of $y' + P(x)y = Q(x)$ in the form $\int Q(t)e^{\int P(t)dt}dt$

☐ true ☐ false $(2Pt.)$

(v) The function $f(t) = t^{-1}$ **is not** piecewise continuous

☐ true ☐ false $(2Pt.)$

(vi) The function $f(t) = e^{\sqrt{t}}$ **is not** of exponential order

☐ true ☐ false $(2Pt.)$

(vii) The function $F(s) = \frac{s}{s+4}$ **is** the Laplace transform of a function that is piecewise continuous and of exponential order

☐ true ☐ false $(2Pt.)$

$(viii)$ $\ell^{-1}\{F(s)G(s)\} \neq \ell^{-1}\{F(s)\}\ell^{-1}\{G(s)\}$

☐ true ☐ false $(2Pt.)$

(Q_2) A mass weighing 8 pounds is attached to a 4-feet-long spring. At equilibrium the spring measures 6 feet. If the mass is initially released from the rest at a point 2 feet below the equilibrium position. Find the displacements $x(t)$ if it is further known that the surrounding medium offers a resistance numerically equal to 2 times the instantaneous velocity. $(10Pt.)$

(Q_3) **Find the general solution of:** $y'' - 2y' + y = e^t tan^{-1} t$ $(10Pt.)$

(Q_4) **Find the general solution of:** $y(x+y+1)dx+(x+2y)dy=0$ $(8Pt.)$

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(Q_5) **Find the Laplace transform**: (i) $\ell\{te^{2t}\sinh 3t\}$, (ii) $\ell\{\cos t\, u(t-\pi)\}$ $(8Pt.)$

(Q_6) **Find the inverse Laplace transform**: $(i)\ \ell^{-1}\{\frac{s}{(s^2+1)^2}\}$, $(ii)\ \ell^{-1}\{\frac{e^{-\pi s}}{s^2+4s+13}\}$ $(10Pt.)$

(Q_7) **Use Laplace transform to solve:** $y''+y=g(t);\ y(0)=1,\ y'(0)=2,$ $(10Pt.)$

$$g(t)=\begin{cases}0 & \text{if } 0\le t\le\pi,\\ \cos t & \text{if } t>\pi.\end{cases}$$

Answer only one of the following two questions:

(Q_8) **Solve:** $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$; $y(0) = \frac{\sqrt{3}}{2}$ $(8Pt.)$

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(Q_9) **Solve:** $y^{\frac{1}{2}} \frac{dy}{dx} + y^{\frac{3}{2}} = x$; $y(0) = 4$ $(8Pt.)$

Q1	**Q2**	**Q3**	**Q4**	**Q5**	**Q6**	**Q7**	**Q8**	**Q9**	**Sum**	**Bal.**