MODEL: B

KING ABDULAZIZ UNIVERSITY

DEPARTMENT OF MATHEMATICS

Exam/Course: Final Exam - Math-204

Student Name: **Student University Number:**

Instructor Name: **Section:**

Time Allowed: 120 Minutes **Jan. 20, 2011**

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$(Q1)$ **Select the correct response:**

(i) The D.E. $(x^2+4)\,dy = (2x - 8xy^2)\,dx$ is

☐ exact ☐ homogeneous ☐ separable $(2Pt.)$

(ii) The D.E. $(yx^2 - x)dx = dy$ is

☐ Ricatti ☐ linear ☐ Bernoulli $(2Pt.)$

(iii) $y = \pi$ is the unique solution of (IVP): $\frac{dy}{dx} = x^2 \sin y;\ y(0) = \pi$

☐ true ☐ false $(2Pt.)$

(iv) There is a particular solution of $y' + P(x)y = Q(x)$ in the form $\int Q(t)e^{\int P(t)dt}dt$

☐ true ☐ false $(2Pt.)$

(v) The function $f(t) = \frac{\sin 3t}{t}$ **is** piecewise continuous

☐ true ☐ false $(2Pt.)$

(vi) The function $f(t) = \cos t$ **is not** of exponential order

☐ true ☐ false $(2Pt.)$

(vii) The function $F(s) = \ln \frac{s^2}{s^2+4}$ **is** the Laplace transform of a function that is piecewise continuous and of exponential order

☐ true ☐ false $(2Pt.)$

$(viii)$ $\ell^{-1}\{F(s)G(s)\} = f(t) * g(t)$

☐ true ☐ false $(2Pt.)$

(Q_2) A mass weighing 24 pounds is attached to a 8-feet-long spring. At equilibrium the spring measures 14 feet. If the mass is initially released from the equilibrium position with an upward velocity 3 feet per second. Find the displacements $x(t)$ if it is further known that the surrounding medium offers a resistance numerically equal to 4 times the instantaneous velocity. $(10Pt.)$

(Q_3) **Find the general solution of:** $y''-4y=\frac{e^{2x}}{x}$ $(10Pt.)$

(Q_4) **Find the general solution of:** $(y^2 - x)\,dx + xy\,dy = 0$ $(8Pt.)$

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(Q_5) **Find the Laplace transform**: (i) $\ell\{te^{-t}\cos t\}$, (ii) $\ell\{\int_0^t \sin\tau\ \cos(t-\tau)d\tau\}$ $(8Pt.)$

(Q_6) **Find the inverse Laplace transform**: $(i)\ \ell^{-1}\{\frac{se^{-\pi}}{s^2+2s+10}\}$, $(ii)\ \ell^{-1}\{\frac{s}{(s^2+1)^2}\}$ $(10Pt.)$

(Q_7) **Use Laplace transform to solve:** $\frac{d^2x}{dt^2}+\omega^2 x = F_0 \cos\omega t;\ x(0)=1,\ x'(0)=1,$ $(10Pt.)$

Answer only one of the following two questions:

(Q_8) **Solve:** $(e^x + e^{-x})\frac{dy}{dx} = y^2$ (8$Pt.$)

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(Q_9) **Solve:** $t\frac{dy}{dt} + y = \frac{1}{y^2}$; $y(0) = 0$ (8$Pt.$)

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Sum	Bal.