MODEL: *A*

KING ABDULAZIZ UNIVERSITY

DEPARTMENT OF MATHEMATICS

Exam/Course: Final Exam - Math-204

Student Name: **Student University Number:**

Instructor Name: **Section:**

Time Allowed: 120 Minutes **Jan. 20, 2011**

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($Q1$) **Select the correct response:**

(i) The D.E. $\frac{dy}{dx} = \frac{y^2-x^2}{x^2+y^2}$ is

☐ exact ☐ homogeneous ☐ separable ($2Pt.$)

(ii) The D.E. $dx = (xy^2 - y)dy$ is

☐ Ricatti ☐ linear ☐ Bernoulli ($2Pt.$)

(iii) $y = 1$ is the unique solution of (IVP): $\frac{dy}{dx} = y \ln y;\ y(0) = 1$

☐ true ☐ false ($2Pt.$)

(iv) There is a particular solution of $y' + P(x)y = Q(x)$ in the form $\int Q(t)e^{\int P(t)dt}dt$

☐ true ☐ false ($2Pt.$)

(v) The function $f(t) = \frac{\sin t}{t}$ **is** piecewise continuous

☐ true ☐ false ($2Pt.$)

(vi) The function $f(t) = e^{t^2}$ **is** of exponential order

☐ true ☐ false ($2Pt.$)

(vii) The function $F(s) = \frac{s^2}{s^2+4}$ **is not** the Laplace transform of a function that is piecewise continuous and of exponential order

☐ true ☐ false ($2Pt.$)

($viii$) $\ell^{-1}\{F(s)G(s)\} = \ell^{-1}\{F(s)\}\ell^{-1}\{G(s)\}$

☐ true ☐ false ($2Pt.$)

(Q_2) A mass weighing 16 pounds is attached to a 5-feet-long spring. At equilibrium the spring measures 8.2 feet. If the mass is initially released from the equilibrium position with an upward velocity 3 feet per second. Find the displacements $x(t)$ if it is further known that the surrounding medium offers a resistance numerically equal to the instantaneous velocity. ($10Pt.$)

(Q_3) **Find the general solution of:** $y''+2y'+y = e^{-t}tan^{-1}t$ ($10Pt.$)

(Q_4) **Find the general solution of:** $(2y^2+3x)\,dx+2xy\,dy=0$ ($8Pt.$)

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(Q_5) **Find the Laplace transform**: (i) $\ell\{te^{2t}\cos 3t\}$, (ii) $\ell\{\int_0^t \sin\tau\ \sin(t-\tau)d\tau\}$ ($8Pt.$)

(Q_6) **Find the inverse Laplace transform**: $(i)\ \ell^{-1}\{\frac{(s-1)e^{-\pi}}{s^2+2s+10}\}$, $(ii)\ \ell^{-1}\{\frac{1}{(s^2+1)^2}\}$ $(10Pt.)$

(Q_7) **Use Laplace transform to solve:** $\frac{d^2x}{dt^2}+\omega^2 x = F_0 \sin \omega t;\ x(0) = 1,\ x'(0) = 1,$ $(10Pt.)$

Answer only one of the following two questions:

(Q_8) **Solve:** $(1+x^4)dy+x(1+4y^2)dx = 0;\ y(1) = 0$ $(8Pt.)$

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(Q_9) **Solve:** $t^2\frac{dy}{dt}+y^2 = ty;\ y(1) = 1$ $(8Pt.)$

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Sum	Bal.